5.2A WS Name Key Date ___________ Per _____

To establish each identity, manipulate the left side of the equation only. Label which identity type used.

Simple Identities

1. $\csc\theta \cdot \cos\theta = \cot\theta$

$\frac{1}{\sin} \cdot \cos$

$\frac{\cos}{\sin}$

$\boxed{\cot}$ ✓

2. $\cos\theta(\tan\theta + \sec\theta) = \sin\theta + 1$

$\cos\left(\frac{\sin}{\cos} + \frac{1}{\cos}\right)$

$\frac{\sin\cdot\cancel{\cos}}{\cancel{\cos}} + \frac{\cancel{\cos}}{\cancel{\cos}}$

$\boxed{\sin + 1}$ ✓

3. $(sec\,\theta + \tan\theta)(sec\,\theta - \tan\theta) = 1$

$(u+v)(u-v)$

$u^2 - v^2$

$\sec^2 - \tan^2$

$1 + \cancel{\tan^2} - \cancel{\tan^2}$

$\boxed{1}$ ✓

4. $\sec\theta \cdot \sin\theta = \tan\theta$

$\frac{1}{\cos} \cdot \sin$

$\frac{\sin}{\cos}$

$\boxed{\tan}$ ✓

5. $1 - \tan^2(-\theta) = \sec^2\theta$

$1 - (\tan^2(-\theta))$

$1 - (-\tan^2\theta)$

$1 + \tan^2$

$\boxed{\sec^2}$ ✓

6. $\tan^2\left(\frac{\pi}{2} - \theta\right) + 1 = \csc^2\theta$

$\cot^2 + 1$

$\boxed{\csc^2}$

Splitting fractions

7. $\frac{1+\cos^2 x}{\cos x} = \sec x + \cos x$

$\frac{1}{\cos} + \frac{\cos^{\cancel{2}}}{\cancel{\cos}}$

$\boxed{\sec + \cos}$ ✓

8. $\frac{\cos^2\theta - \sin\theta}{sin^2\theta} = cot^2\theta - \csc\theta$

$\frac{\cos^2}{\sin^2} - \frac{\sin}{\sin^2}$

$\cot^2 - \frac{1}{\sin}$

$\boxed{\cot^2 - \csc}$ ✓

Splitting fractions (continued)

9. $\frac{\cos^2 x+\csc x}{\sin^2 x} = \cot^2 x + csc^3\, x$

$\frac{\cos^2}{\sin^2} + \frac{\csc}{\sin^{\cancel{2}}\cancel{x}}$

$\boxed{\cot^2 + \csc^3}$

$\csc \div \sin^2$

$\frac{1}{\sin} \cdot \frac{1}{\sin^2}$

$\frac{1}{\sin^3}$

10. $\frac{\sec^2 \beta-\tan^2 \beta+\tan \beta}{\sec \beta} = \sin\beta + \cos\beta$

$1 + \tan^2$

$1 + \cancel{\tan^2} - \cancel{\tan^2} + \tan$

$\frac{1+\tan}{\sec}$

$\frac{1}{\sec} + \frac{\tan}{\sec}$

$\tan \div \frac{1}{\cos}$

$\frac{\sin}{\cancel{\cos}} \cdot \frac{\cancel{\cos}}{1}$

$\boxed{\cos + \sin}$

Using Conjugates

11. $\frac{\cos\theta}{1+\sin\theta} = sec\theta(1 - sin\theta)$

$\frac{\cos}{1+\sin} \cdot \frac{1-\sin}{1-\sin}$

$\frac{\cos(1-\sin)}{1-\sin^2} \rightarrow \cos^2 + \cancel{\sin^2} - \cancel{\sin^2}$

$\frac{\cancel{\cos}(1-\sin)}{\cos^{\cancel{2}}}$

$\frac{1-\sin}{\cos} \rightarrow \frac{1}{\cos}\left(\frac{1-\sin}{1}\right)$

$\boxed{\sec \cdot (1-\sin)}$

12. $1 - \frac{\sin^2 x}{1-\cos x} = -\cos x$

$\frac{1-\cos}{1-\cos} - \frac{\sin^2}{1-\cos}$

$\frac{1-\cos^{\cancel{3}} - \sin^2}{1-\cos}$

$\frac{\cos^2 - \cos \cancel{- \sin^2} \cancel{+\sin^2}}{1-\cos}$

$\frac{\cos(\cos-1)\cdot -1}{1-\cos}$

$\frac{-\cos(1-\cancel{\cos})}{(\cancel{1-\cos})}$

$\boxed{-\cos}$

5.2B WS Name ____________________ Date __________ Per _____

To establish each identity, manipulate only one side of the equation. Label which identity type used.

7) 1. $\sec^4\theta - \sec^2\theta = \tan^4\theta + tan^2\theta$

$\sec^2(\sec^2-1)$	Factor out $\sec^2$
$(1+\tan^2)(\tan^2)$	Pythag Identities
$\tan^2+\tan^4$	Distribute
$\tan^4+\tan^2$	Commutative Prop

21) 2. $3sin^2\theta + 4cos^2\theta = 3 + cos^2\theta$

$3(1-\cos^2)+4\cos^2$	Pythag.
$3-3\cos^2+4\cos^2$	Distribute
$3+\cos^2$	Combine Like Terms

31) 3. $\frac{1-sin\alpha}{cos\alpha}\left(\frac{1-\sin\alpha}{1-\sin\alpha}\right) + \frac{cos\alpha}{1-sin\alpha}\left(\frac{\cos}{\cos}\right) = 2sec\,\alpha$

$\frac{\cos-\sin\cancel{\cos}}{\cos^2}+\frac{\cos+\sin\cancel{\cos}}{\cos^2}$	Get a common denom of $\cos^2\alpha$ Left × cos Right × conjugate
$\frac{2\cos}{\cos^2}$	Combine Like Terms
$\frac{2}{\cos}$	Reduce
$2\sec$	Reciprocal

62) 4. $\frac{sin\alpha+cos\alpha}{sin\alpha} - \frac{cos\alpha-sin\alpha}{cos\alpha} = sec\,\alpha\ csc\,\alpha$

$\frac{\sin\cancel{\cos}+\cos^2}{\sin\cos}-\frac{\cos\cancel{\sin}-\sin^2}{\sin\cos}$	common denom. of sincos
$\frac{\cos^2+\sin^2}{\sin\cos}$	Combine Like Terms
$\frac{1}{\sin\cos}$	Pythag Identity
$\sec\csc$	Reciprocal

36 5. $\frac{1-2cos\theta+cos^2\theta}{sin^2\theta} = (csc\theta - cot\theta)^2$

$\frac{1}{sin^2} - \frac{2cos}{sin^2} + \frac{cos^2}{sin^2}$	Split into 3 fractions
$csc^2 - 2cotcsc + cot^2$	Reciprocal & Quotient Identities
$(csc - cot)^2$	Factor

40 6. $\frac{sin\theta cos\theta}{cos^2\theta - sin^2\theta} = \frac{tan\theta}{1-tan^2\theta}$

$\frac{sincos/cos^2}{cos^2/cos^2 - sin^2/cos^2}$	Divide every term by cos^2
$\frac{tan}{1-tan^2}$	Quotient Identities

06 7. $\frac{cosx+sinx-sin^3x}{sinx} = cotx + cos^2x$

$\frac{cos}{sin} + \frac{sin}{sin} - \frac{sin^3}{sin}$	Split into 3 fractions
$cot + 1 - sin^2$	Quotient Identity
$cot + cos^2$	Pythag Identity

Bonus:

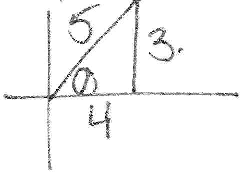
67 8. $\frac{(2cos^2\theta-1)^2}{cos^4\theta - sin^4\theta} = 1 - 2\sin^2\theta$

$\frac{(2cos^2 - sin^2 - cos^2)^2}{(cos^2+sin^2)(cos^2-sin^2)}$	Pythag Identity & Factoring
$\frac{(cos^2-sin^2)^{\cancel{2}}}{1\,\cancel{(cos^2+sin^2)}\cancel{(cos^2-sin^2)}}$	Combine Like Terms, Reduce fraction, Pythag Identity
$cos^2 - sin^2$	
$1 - sin^2 - sin^2$	Pythag Identity
$1 - 2sin^2$	Combine Like Terms

5.1-5.2 Review WS Name Danielle Per ______

Evaluate without using a calculator. Use your identities so solve them not triangles.

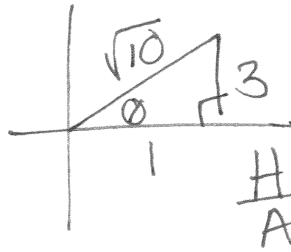
1. Find $\sin\theta$ and $\cos\theta$ if $\tan\theta = \frac{3}{4}$ (O over A) and $\sin\theta > 0$



$\sin = 3/5$

$\cos = 4/5$ $\frac{A}{H}$

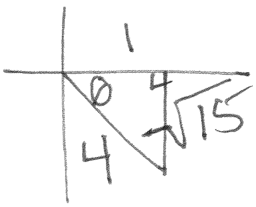
2. Find $\sec\theta$ and $\csc\theta$ if $\tan\theta = \frac{3}{1}$ and $\cos\theta > 0$



$\frac{H}{A} = \sec = \frac{\sqrt{10}}{1}$

$\frac{H}{O} = \csc = \frac{\sqrt{10}}{3}$

3. find $\tan\beta$ and $\cot\beta$ if $\sec\beta = \frac{4}{1}$ and $\sin\beta < 0$ $\frac{H}{A}$



$\tan = \frac{-\sqrt{15}}{1}$

$\cot = -\frac{\sqrt{15}}{15}$

4. Find $\sin\theta$ and $\tan\theta$ if $\cos\theta = 4/5$ and $\tan\theta < 0$

Verify the Identities.

5. $(\cos x)(\tan x + \sin x \cot x) = \sin x + \cos^2 x$

$\cancel{\cos} \cdot \frac{\sin}{\cancel{\cos}} + \cos \cdot \cancel{\sin} \cdot \frac{\cos}{\cancel{\sin}}$

$\sin + \cos^2$

✓

6. $(1-\tan x)^2 = \sec^2 x - 2\tan x$

$1 - 2\tan + \tan^2$

$1 + \tan^2 = \sec^2$

$\sec^2 - 2\tan$

✓

7. $\frac{(1-\cos x)(1+\cos x)}{\cos^2 x} = \tan^2 x$

$$\frac{1-\cos^2}{\cos^2} \Rightarrow \frac{\sin^2}{\cos^2}$$

$\tan^2$ ✓

8. $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$

$$\frac{1+\cancel{\cos x}+1-\cancel{\cos x}}{1-\cos^2 x}$$

$$\frac{2}{1-\cos^2 x} = \frac{2}{\sin^2 x}$$

$2\csc^2$ ✓

9. $\frac{\sec x+1}{\tan x} = \frac{\sin x}{1-\cos x}$

$$\frac{\sec}{\tan} + \frac{1}{\tan x}$$

$$\frac{1}{\cos} \div \frac{\sin}{\cos}$$

$$\frac{1}{\cancel{\cos}} \cdot \frac{\cancel{\cos}}{\sin}$$

$$\frac{1}{\sin} + \cancel{\frac{1}{\tan}}\ \frac{\cos}{\sin}$$

$$\frac{1+\cos}{\sin} \cdot \frac{1-\cos}{1-\cos}$$

$1-\cos^2 = \frac{\sin^2}{\cancel{\sin}(1-\cos} = \frac{\sin}{1-\cos}$ ✓

10. $\cot^2 x - \cos^2 x = \cos^2 x \cot^2 x$

$u^2 - v^2$

$(u+v)(u-v)$

$\left(\frac{\cos}{\sin} + \frac{\cos \cdot \sin}{\sin}\right)\left(\frac{\cos}{\sin} - \frac{\cos \cdot \sin}{\sin}\right)$

$\left(\frac{\cos(1+\sin)}{\sin}\right)\left(\frac{(\sin-1)\cos}{\sin}\right)$

$(1+x)(x-1) \to (1-x)$

$$\frac{\cos^2(1+\sin)(\sin-1)}{\sin^2}$$

$\cot^2$

$\cot^2(1-\sin^2)$

$\cot^2\cos^2$ ✓

5.3C Worksheet Name ________________________ Per ____

Prove the identities.

1. $(\cos x-\sin x)^2=1-2\sin x\cos x$

2. $\tan x+\sec x=\frac{\cos x}{1-\sin x}$

3. $\frac{\sec^2 x-1}{\sin x}=\frac{\sin x}{1-\sin^2 x}$

4. $\tan^2 x-\sin^2 x=\tan^2 x\sin^2 x$

5. $\frac{\sin t}{1-\cos t}+\frac{1+\cos t}{\sin t}=\frac{2(1+\cos t)}{\sin t}$

6. $\sin^3 x\cos^3 x=(\sin^3 x-\sin^5 x)(\cos x)$

7. $\frac{1-3\cos x-4\cos^2 x}{\sin^2 x}=\frac{1-4\cos x}{1-\cos x}$

8. $\frac{1}{\tan\beta}+\tan\beta=\sec\beta\csc\beta$

Find all solutions on the interval of $[0,2\pi)$.

9. $2\cos x\sin x-\cos x=0$

10. $\sqrt{2}\tan x\cos x - \tan x = 0$

11. $3\cot^2 x-1=0$

12. $\sec x\csc x=2\csc x$

Find all solutions.

13. $4\cos^2 x-4\cos x+1=0$

14. $3\sin t=2\cos^2 t$

15. $2\sin^2 x+3\sin x+1=0$

16. $2\cos 2x=1$

17. $\cos 2x = -1$

18. $\tan^2 3x = 3$

19. $\cos 2x(2\cos x+1)=0$

20. $\tan\frac{x}{3} = 1$

21. $\csc^2 2x=3\csc 2x+4$

Use a graphing utility to approximate the solutions(to three decimal places) on the interval $[0,2\pi)$.

22. $2\sin x + \cos x = 0$

23. $\csc^2 x + 0.5\cot x = 5$

24. $4\cos^2 x - 2\sin x + 1=0$

5.3C

1. $(\cos x - \sin x)^2 = 1 - 2\sin x \cdot \cos x$

$$\underbrace{\cos^2 - 2\cos\sin + \sin^2}_{1}$$

$1 - 2\sin \cdot \cos$ qed ✓

2. $\frac{\cos x}{1 - \sin x} = \tan x + \sec x$

$$\frac{\cos \cdot (1 + \sin x)}{1 - \sin^2 x} = \frac{\cancel{\cos}(1 + \sin)}{\cos^{\cancel{2}}}$$

$\frac{1}{\cos} + \frac{\sin}{\cos} = \sec x + \tan x$ ✓

3. $\frac{\sec^2 - 1}{\sin} = \frac{\sin x}{1 - \sin^2}$

$$\frac{\sec^2 - 1}{\sin} = \frac{\tan^2}{\sin}$$

$$\frac{\sin^2}{\cos^2} \div \sin \Rightarrow \frac{\sin^{\cancel{2}}}{\cos^2} \cdot \frac{1}{\cancel{\sin}}$$

$$\frac{\sin}{\cos^2} \cdot \cancel{\frac{1}{\cos}} = \frac{\sin x}{1 - \sin^2}$$ ✓

4. $\tan^2 - \sin^2 = \tan^2 \sin^2 x$

$(\tan + \sin)(\tan - \sin)$

$\left(\frac{\sin}{\cos} + \frac{\sin \cdot \cos}{\cos}\right)\left(\frac{\sin}{\cos} - \frac{\sin \cdot \cos}{\cos}\right)$

$\left(\frac{\sin + \sin\cos}{\cos}\right)\left(\frac{\sin - \sin\cos}{\cos}\right)$

$\frac{\sin(1+\cos)\ \sin(1-\cos)}{\cos^2}$

$\frac{\sin^2(1-\cos^2)}{\cos^2} = \tan^2 \cdot \sin^2$ ✓

5. $\frac{\sin}{1-\cos} + \frac{1+\cos}{\sin}$

$\frac{\sin^2 + (1-\cos^2)}{\sin(1-\cos)} = \frac{\sin^2 + \sin^2}{\sin(1-\cos)}$

$\frac{2\sin^{\cancel{2}}}{\cancel{\sin}(1-\cos)} \cdot \frac{(1+\cos)}{(1+\cos} = \frac{2\sin(1+\cos)}{(1-\cos^2)}$

$\frac{2\cancel{\sin}(1+\cos)}{\sin^{\cancel{2}}} = \frac{2(1+\cos)}{\sin}$ ✓

6. $\sin^3 \cdot \cos^3 = (\sin^3 - \sin^5)\cos$

$\sin^3(\cancel{s}\, 1 - \sin^2)\cos$

$\sin^3(\cos^2)\cos = \sin^3 \cdot \cos^3$ ✓

7. $\dfrac{1 - 3\cos x - 4\cos^2}{\sin^2} = \dfrac{-1(4u^2 + 3u - 1)}{1 - \cos^2}$

$4u^2 + 4u - u - 1 = \dfrac{-(4u-1)(u+1)}{1-\cos^2}$

$4u \quad -1$

$\dfrac{-1(4\cos - 1)(\cos + 1)}{1 - \cos^2} = \dfrac{(-4\cos + 1)\cancel{(\cos + 1)}}{\cancel{(\cos + 1)}(\cos - 1)}$

$\downarrow$

$-1(\cos^2 - 1)$

$\boxed{\dfrac{1 - 4\cos x}{1 - \cos x}}$ ✓

8. $\dfrac{1}{\tan} + \tan = \sec \cdot \csc$

$\dfrac{1}{\tan} + \dfrac{\tan^2}{\tan} = \dfrac{1 + \tan^2}{\tan} = \dfrac{\sec^2}{\tan}$

$\dfrac{1}{\cos^2} \div \dfrac{\sin}{\cos} \rightarrow \dfrac{1}{\cos^{\cancel{2}}} \circ \dfrac{\cancel{\cos}}{\sin}$

$\dfrac{1}{\cos \cdot \sin} = \sec x \cdot \csc x$ ✓

9. $2\cos \cdot \sin - \cos = 0$

$(2\sin - 1)\cos = 0$

$2\sin - 1 = 0$ $\qquad$ $\cos x = 0$

$\sin x = 1/2$

$\boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$ $\qquad$ $@, \boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

10. $\sqrt{2} \cdot \tan \cdot \cos x - \tan x = 0$

$\tan(\sqrt{2} \cdot \cos - 1) = 0$

$\tan = 0$ $\qquad$ $\cos = 1/\sqrt{2} = \sqrt{2}/2$

$@\boxed{0, \pi}$ $\qquad$ $@\boxed{\frac{\pi}{4} \ \frac{5\pi}{4}}$

11. $3 \cdot \cot^2 - 1 = 0$

$\sqrt{\cot^2} = \sqrt{1/3}$ $\qquad$ $\cot = \sqrt{3}/3$

$\tan = \sqrt{3} = \boxed{\frac{\pi}{3}, \frac{4\pi}{3}}$

12. $\sec \cdot \csc = 2\csc$

$\sec \cdot \csc - 2\csc = 0$

$\csc(\sec - 2) = 0$

$\csc = 0$ $\qquad$ $\sec = 2$

$\frac{1}{\sin} = \frac{0}{\#}$ $\qquad$ $\cos = 1/2$

$\sin = \text{undef}$ $\qquad$ $\boxed{\frac{\pi}{3} \quad \frac{5\pi}{3}}$

13. $4\cos^2 - 4\cos + 1$ $\qquad$ $4u^2 - 4u + 1$

$(2\cos - 1)^2 = 0$ $\qquad$ $(2u-1)^2$

$2\cos = 1$

$\cos x = 1/2$

$\boxed{\frac{\pi}{3} \pm 2\pi n \quad \frac{5\pi}{3} \pm 2\pi n}$

14. $3\sin = 2\cos^2$

$3\sin = 2(1 - \sin^2)$

$2\sin^2 + 3\sin - 2$

$(\sin + 2)(2\sin - 1)$

$\hookrightarrow$ DNE

$\sin = \frac{1}{2}$

$\frac{\pi}{6} \pm 2\pi n$

$\frac{5\pi}{6} \pm 2\pi n$

5.3

15. $2u^2+3u+1=0$

$(u+1)(2u+1)$

$(\sin+1)(2\sin+1)$

$\sin x=-1$ $\qquad$ $\sin x = -1/2$

$\boxed{\frac{3\pi}{2} \pm 2\pi n}$ $\qquad$ $\boxed{\begin{array}{l}\frac{7\pi}{6} \pm 2\pi n \\ \frac{11\pi}{6} \pm 2\pi n\end{array}}$

16. $2\cos 2x=1$ $\qquad$ $\cos 2x = \frac{1}{2}$

$2(\cos^2-\sin^2)=1$ or $\qquad$ $\cos x = \frac{1}{2}$

$\cos^2 - \sin^2 = 1/2$ $\qquad$ @ $\frac{\pi}{3}$ & $\frac{5\pi}{3}$

$x=\frac{\pi}{6} \pm 2\pi n, \frac{5\pi}{6} \pm 2\pi n$

17. $\cos 2x = -1$

$\cos x = -1$

@ π

so $\boxed{\frac{\pi}{2} \pm 2\pi n}$

18. $\tan^2 3x = 3$

$\tan 3x = \pm\sqrt{3}$

every $\frac{\pi}{3} \cdot \frac{1}{3}$

$\boxed{\frac{\pi}{9} \pm \frac{\pi}{2} n}$

5.3D WS Name Danielle Date ____________ Per ______

Solve the following equations on the interval $[0, 2\pi]$.

1. $\cos\frac{x}{4} = 0$

$\cos x = 0$ @ $\frac{\pi}{2}$, $\frac{3\pi}{2}$

$\frac{x}{4} = \frac{\pi}{2}$ $\boxed{x = 2\pi}$

$\frac{x}{4} = \frac{3\pi}{2}$ $\boxed{x = 6\pi}$

2. $\cos 3x + 1 = \sin 3x$

y_1 y_2

$\boxed{\text{intersect @ } y = 1, 0}$

$\cos 3x + 1 = 1$

$\cos 3x = 0$

$x = \cancel{\frac{\pi}{6}}, \boxed{\frac{\pi}{2}}$

$\sin 3x = 1$

@ $\frac{\pi}{2}$

$3x = \frac{\pi}{2}$

$\boxed{\frac{\pi}{6}}$

all $\boxed{x = \cancel{0}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}}$

3. $2\sin^2 2x = 1$

$\sin^2(2x) = \frac{1}{2}$

$\sin(2x) = \frac{\pm\sqrt{2}}{2}$

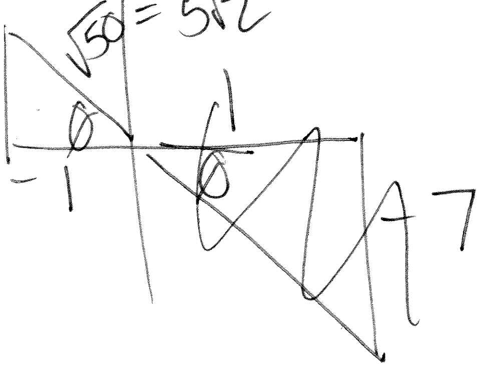
@ $\frac{\pi}{4}$ $\frac{3\pi}{4}$ $\frac{5\pi}{4}$ $\frac{7\pi}{4}$

$\boxed{x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}}$

4. $\cos 2x\,(2\cos x + 1) = 0$

5. Find all trig functions using identities, given $\tan\theta = \frac{-7}{1}$ and $\sin\theta > 0$.

$\sqrt{50} = 5\sqrt{2}$



$\sin\theta = \frac{7}{5\sqrt{2}}$

$\cos\theta = {-1}/{5\sqrt{2}}$

$\tan\theta = -7$

$\csc\theta = \frac{5\sqrt{2}}{7}$

$\sec\theta = {-1}/\cancel{\ \ }\ 5\sqrt{2}$

$\cot\theta = {-1}/{7}$

Find all solutions for the following equations.

6. $\sin\frac{x}{2} = 0$

see notes

$\boxed{0 \pm 2\pi n}$

7. $\sec 4x = 2$

$\frac{1}{\cos} = \sec$

$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}}$

see $\cos x = \frac{1}{2}$ @ $\frac{\pi}{3}$ $\frac{5\pi}{3}$

8. $\tan\frac{x}{3} = 1$

$\tan x = 1$ @ $\frac{\pi}{4}$ $\frac{5\pi}{4}$

$\boxed{x = \frac{3\pi}{4}, \cancel{\frac{15\pi}{4}} \pm 2\pi n}$

9. $\underset{\uparrow}{\sec^2} 3x + \tan 3x = 1$

$\tan^2 + 1$

$\tan^2(3x) + \tan(3x) = 0$

$\tan 3x(\tan(3x) + 1) = 0$

$\tan x = 0$ $\quad$ $\tan x = -1$

$0, \pi$ $\quad$ $\frac{3\pi}{4}$ $\frac{7\pi}{4}$

$$\boxed{\begin{aligned} &\tfrac{21\pi}{4} \pm 2\pi n \\ x = \ &0 \pm 2\pi n \\ &\tfrac{\pi}{3} \pm 2\pi n \\ &\tfrac{3\pi}{4} \pm 2\pi n \end{aligned}}$$

2\. cont'd

$\cos 3x = -1$

$\cos x = -1$ @ π

$3x = \pi$

$\boxed{x = \frac{\pi}{3}}$

sin

$\cancel{\cos} 3x = 0$

$\sin x = 0$ @ $0, \pi$

$\cancel{\sin}$ $3x = 0$

$\boxed{x = 0}, \frac{\pi}{3}$

dup

5.3E Worksheet Name Danielle Per ____

Prove the identities.

1. $\overset{u}{tan^2x} - \overset{v}{sin^2x} = tan^2x sin^2x$

$(u+v)(u-v)$

2. $\frac{sint}{1-cost} + \frac{1+cost}{sint} = \frac{2(1+cost)}{sint}$

old hw

3. $tan \propto + sec \propto = \frac{cos\propto}{1-sin\propto}$

see paper

4. $\frac{1-3cosx-4cos^2x}{sin^2x} = \frac{1-4cosx}{1-cosx}$

old hw

Find all solutions on the interval of [0,2π).

5. 2cosxsinx-cosx=0

$\cos(2\sin-1)=0$

$\cos x = 0$ $\sin x = \frac{1}{2}$

@ $\frac{\pi}{2}, \frac{3\pi}{2}$ @ $\frac{\pi}{6}$ $\frac{5\pi}{6}$

6. $\sqrt{2}tanxcosx - tanx = 0$

$\tan x(\sqrt{2}\cos x - 1) = 0$

$\tan x = 0$ $\cos x = \frac{\sqrt{2}}{2}$

@ $0, \pi$

@ $\frac{\pi}{4}$ $\frac{7\pi}{4}$

$x=\frac{1}{2}$ $2x-1=0$ $\sin^2+\cos^2=1$

Find all solutions. If the answer is not on the unit circle round your answer to the nearest hundredth.

7. $4\cos^2x-4\cos x+1=0$

$(2x-1)^2$

$(2\cos x-1)^2=0$

$\cos x=\frac{1}{2}$

$\boxed{x=\frac{\pi}{3},\frac{5\pi}{3}}$

8. $3\sin t=2\cos^2t$

$3\sin=2(1-\sin^2)$

$2\sin^2+3\sin-2$

$(\sin+2)(2\sin-1)$

$\sin x=-2$ $\sin=\frac{1}{2}$

$\boxed{\text{DNE}}$ $\boxed{x=\frac{\pi}{6}\ \frac{5\pi}{6}}$

9. $2\sin^2x+3\sin x+1=0$

$\sin x=-1$ $\sin x=\frac{-1}{2}$

$\boxed{\frac{3\pi}{2}}$ $\boxed{\frac{7\pi}{6}\ \frac{11\pi}{6}}$

10. $2\cos 2x=1$

$\cos(2x)=\frac{1}{2}$

$2x=\frac{\pi}{3},\frac{5\pi}{3}$

$\boxed{x=\frac{\pi}{6}\ \frac{5\pi}{6}}$

11. $\tan\frac{x}{3}=1$

$\tan x=1$ @ $\frac{\pi}{4}\ \frac{5\pi}{4}$

$\boxed{x=\frac{3\pi}{4},\frac{15\pi}{4}}$

12. $\csc^2 2x=3\csc 2x+4$

$\csc^2-3\csc-4$

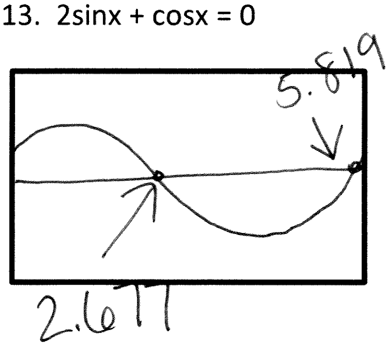
$\csc 2x=-1$ $\csc 2x=4$

$2\cancel{x}=\frac{3\pi}{2}$

$\boxed{x=\frac{3\pi}{4}}$

Use a graphing utility to approximate the solutions(to three decimal places) on the interval [0,2π).

13. $2\sin x + \cos x = 0$



14. $\csc^2x + 0.5\cot x = 5$

15. $4\cos^2x – 2\sin x + 1=0$

16. Find the remaining five trig functions if $sec\theta=\frac{3}{2}$ and $sin\theta>0$ using only identities.

5.4B Worksheet Name Key Per ______

Find the exact value of each trigonometric function.

1. $sin\frac{5\pi}{12}$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

2. $cos\frac{7\pi}{12}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

3. $\tan 15^\circ$

$\frac{\sqrt{3}-1}{\sqrt{3}+1}$

4. $\sec(-\frac{\pi}{12})$

$\frac{4}{\sqrt{6}+\sqrt{2}}$

Find the exact values for each expression.

5. $\sin 20^\circ \cos 10^\circ + \cos 20^\circ \sin 10^\circ$

sin 30

$\frac{1}{2}$

6. $\cos 40^\circ \cos 10^\circ + \sin 40^\circ \sin 10^\circ$

cos 30

$\frac{\sqrt{3}}{2}$

7. $\frac{tan40^o - tan10^o}{1+tan40^o tan10^o}$

tan 30

$\frac{\sqrt{3}}{3}$

Find the exact value of each of the following under the given conditions.

8. Find sin(x+y) if sinx=3/5, 0<x<π/2; cosy=$\frac{2\sqrt{5}}{5}$, $-\frac{\pi}{2} < y < 0$

$\frac{2\sqrt{5}}{25}$

9. Find cos(x-y) if cosx=$\frac{\sqrt{5}}{5}$, $o < x < \frac{\pi}{2}$; siny= -4/5 , $-\frac{\pi}{2} < y < 0$

$-\frac{\sqrt{5}}{5}$

10. Find tan(x+y) if tanx = -4/3, $\frac{\pi}{2} < x < \pi$; cosy=1/2, $o < x < \frac{\pi}{2}$

$\frac{-4+3\sqrt{3}}{3+4\sqrt{3}}$

Verify the identities.

11. $\sin(\frac{\pi}{2} + \theta) = cos\theta$

12. $sin\theta csc\theta - cos^2\theta = sin^2\theta$

13. $1 - \frac{sin^2\theta}{1+cos\theta} = cos\theta$

14. $\frac{\sin(x+y)}{\sin(x-y)} = \frac{tanx+tany}{tanx-tany}$

15. $sec^4\theta - sec^2\theta = tan^4\theta + tan^2\theta$

16. $\cos\left(\frac{\pi}{2}+\theta\right) = -sin\theta$

Solve each equation on the interval 0≤θ<2π.

17. $2cos^2\theta + cos\theta = 0$

$\theta = \frac{\pi}{2}\ \frac{3\pi}{2}\ \frac{2\pi}{3}\ \frac{4\pi}{3}$

18. $\tan\left(\frac{\theta}{2}+\frac{\pi}{3}\right) = 1$

$\theta = \frac{11\pi}{6}$

19. 4cosθ=1+2cosθ

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

Use a graphing utility to solve each equation on the interval 0≤θ<2π.

20. 2x = 5cosx

x = 1.11

21. sinx = lnx

2.22

22. 2sinx+3cosx = 4x

.87

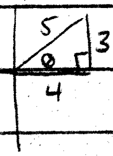
23. Write sin(arcsinx+arccosx) as an algebraic expression.

1

24. Find the value of the expression without a calculator. cos(arcsin(-1)+arccos(0))

1

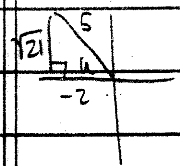
5.5A p394 ~~13~~, 17, ~~39~~, 43, 47, 61, 73, 81, 93-97o

~~13.~~ 

$\sin 2u = 2\sin u\cos u$
$= 2\cdot\frac{3}{5}\cdot\frac{4}{5}$
$= \frac{24}{25}$

$\cos 2u = 1-2\sin^2 u$
$= 1-2\left(\frac{3}{5}\right)^2$
$= 1-2\cdot\frac{9}{25}$
$= 1-\frac{18}{25}$
$= \frac{25}{25}-\frac{18}{25} = \frac{7}{25}$

$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$
$= \frac{2\cdot 3/4}{1-(3/4)^2}$
$= 3/2 \big/ 1-9/16$
$= 3/2 \big/ 7/16$
$= 3/2 \cdot 16/7$
$= \frac{24}{7}$

✓ 17. 

$25-4=21$

$\sin 2u = 2\sin u\cos u$
$= 2\left(\frac{\sqrt{21}}{5}\right)\left(-\frac{2}{5}\right)$
$= \frac{-4\sqrt{21}}{25}$

$\cos 2u = \cos^2 u - \sin^2 u$
$= \left(-\frac{2}{5}\right)^2 - \left(\frac{\sqrt{21}}{5}\right)^2$
$= \frac{4}{25}-\frac{21}{25}$
$= \frac{-17}{25}$

$\tan 2u = \frac{2\tan u}{1-\tan^2 u}$
$= \frac{2(\sqrt{21}/-2)}{1-(\sqrt{21}/-2)^2}$
$= \frac{2\sqrt{21}}{-2} \Big/ 1-\frac{21}{4}$
$= \frac{2\sqrt{21}}{+2}\cdot\frac{+4}{17}$
$= 4\sqrt{21}/17$

~~39.~~ $\sin\left(\frac{30}{2}\right) = +\sqrt{\frac{1-\cos 30}{2}}$
$= \sqrt{\frac{1-\sqrt{3}/2}{2}}$
$= \sqrt{\frac{2-\sqrt{3}}{4}}$
$= \frac{\sqrt{2-\sqrt{3}}}{2}$

$\cos\left(\frac{30}{2}\right) = +\sqrt{\frac{1+\cos 30°}{2}}$
$= \sqrt{\frac{1+\sqrt{3}/2}{2}}$
$= \sqrt{\frac{2+\sqrt{3}}{4}}$
$= \frac{\sqrt{2+\sqrt{3}}}{2}$

$\tan\left(\frac{30}{2}\right) = \frac{1-\cos 30}{\sin 30}$
$= \frac{1-\sqrt{3}/2}{\frac{1}{2}}$
$= 2-\sqrt{3}$

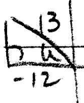
43. $\sin\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}}$
$= \sqrt{\frac{1-\sqrt{2}/2}{2}}$
$= \sqrt{\frac{2-\sqrt{2}}{4}}$
$= \frac{\sqrt{2-\sqrt{2}}}{2}$

$\cos\left(\frac{\pi/4}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}}$
$= \sqrt{\frac{1+\sqrt{2}/2}{2}}$
$= \sqrt{\frac{2+\sqrt{2}}{4}}$
$= \frac{\sqrt{2+\sqrt{2}}}{2}$

$\tan\left(\frac{\pi/4}{2}\right) = \frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}$
$= \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$
$= \frac{2-\sqrt{2}}{\sqrt{2}}$
$= \frac{2\sqrt{2}-1}{2}$

47. $\sin u = \frac{5}{13}$ $\qquad \sin\left(\frac{u}{2}\right) = +\sqrt{\frac{1-\cos u}{2}}$ $\qquad \cos\left(\frac{u}{2}\right) = \sqrt{\frac{1+\cos u}{2}}$ $\qquad \tan\left(\frac{u}{2}\right) = \frac{1-\cos u}{\sin u}$

$\cos u = -\frac{12}{13}$ $\qquad = \sqrt{\frac{1+\frac{12}{13}}{2}}$ $\qquad = \sqrt{\frac{1-\frac{12}{13}}{2}}$ $\qquad = \frac{1+\frac{12}{13}}{5/13}$



$\frac{\pi}{2} < u < \pi$ $\qquad = \sqrt{\frac{\frac{25}{13}}{2}}$ $\qquad = \sqrt{\frac{\frac{1}{13}}{2}}$ $\qquad = \frac{25}{13} \Big/ \frac{5}{13}$

$\boxed{\frac{\pi}{4} < \frac{u}{2} < \frac{\pi}{2}}$ $\qquad = \sqrt{\frac{25}{26}}$ $\qquad = \sqrt{\frac{1}{26}}$ $\qquad = 5$

61. $6\left[\frac{1}{2}\left(\sin\left(\frac{\pi}{3}+\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{3}-\frac{\pi}{3}\right)\right)\right]$

$6\left[\frac{1}{2}\left(\sin\frac{2\pi}{3}+\sin 0\right)\right]$

$3\sin\frac{2\pi}{3}$

73 $\sin 5\theta - \sin\theta$

$2\cos\left(\frac{5\theta+\theta}{2}\right)\sin\left(\frac{5\theta-\theta}{2}\right)$

$2\cos\frac{3\theta}{2}\sin 2\theta$

81. $2\sin\left(\frac{195+105}{2}\right)\cos\left(\frac{195-105}{2}\right)$

$2\sin\left(\frac{300}{2}\right)\cos\left(\frac{90}{2}\right)$

$2\sin 150^\circ \cos 45^\circ$

$2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}}{2}$

93. $\csc 2\theta = \frac{\csc\theta}{2\cos\theta}$

recip $\frac{1}{\sin 2\theta} =$

Double∠ $\frac{1}{2\sin\theta\cos\theta} =$

recip $\frac{\csc\theta}{2\cos\theta} = \frac{\csc\theta}{2\cos\theta}$

95 $\cos^2 2\alpha - \sin^2 2\alpha = \cos 4\alpha$

double∠ $\cos 2(2\alpha) =$

$\cos 4\alpha = \cos 4\alpha$

97. $(\sin x + \cos x)^2 = 1 + \sin 2x$

$\sin^2 x + 2\sin\cos x + \cos^2 x =$

pythag $1 + 2\sin\cos x =$

double∠ $1 + \sin 2x = 1 + \sin 2x$

$$\sin\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1-\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\cos\left(\frac{\frac{\pi}{4}}{2}\right) = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \frac{\sqrt{2-\sqrt{2}}}{2}$$

$$\tan\left(\frac{\frac{\pi}{4}}{2}\right) = \frac{1-\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \frac{2-\sqrt{2}}{\sqrt{2}}$$

$$\csc 2\theta = \frac{\csc\theta}{2\cos\theta}$$

$$\frac{1}{\sin 2\theta} =$$ recip.

$$\frac{1}{2\sin\theta\cos\theta} =$$ double ∢ form.

$$\frac{\csc\theta}{2\cos\theta} = \frac{\csc\theta}{2\cos\theta}$$ reciprocal

$$\cos^2 2\theta - \sin^2 2\theta = \cos 4\theta$$

$$\cos 2(2\theta) =$$

$$\cos 4\theta = \cos 4\theta$$

PreCalc Ch 5 Review WS #1 Name Key Per _______

1. Find all solutions to the equation in the interval $[0,2\pi)$.

a) $4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

b) $\sin x \tan^2 x = \sin x$

$\sin x \tan^2 x - \sin x = 0$

$\sin x(\tan^2 x - 1) = 0$

$\sin x = 0 \quad \tan x = \pm 1$

$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

c) $\sin\left(x+\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$

$\cancel{\sin x\cos\frac{\pi}{6}} + \cos x\sin\frac{\pi}{6} - (\cancel{\sin x\cos\frac{\pi}{6}} - \cos x\sin\frac{\pi}{6}) = \frac{1}{2}$

$\frac{1}{2}\cos x + \frac{1}{2}\cos x = \frac{1}{2}$

$\cos x = \frac{1}{2}$

$x = \pi/3, 5\pi/3$

2. Find all solutions to the equation, using your calculator in the interval $[0,2\pi)$. Round to two decimal places.

a) $\cos 2x + 5\cos x = 2$

$x \approx 1.05, 5.24$

b) $\sin 3x = \sin x$

$x \approx 0, .79, 2.4, 3.9, 5.5$

c) $5\sin 2x - 12\cos 2x = 0$

$x \approx .59, 2.16, 3.73, 5.3$

3. Find the exact value.

a) $\sin\frac{\pi}{12} = \sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)$ or $\sin\left(\frac{\pi/6}{2}\right)$

$\sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

$= \sqrt{\frac{1-\cos\frac{\pi}{6}}{2}}$

$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$= \frac{\sqrt{2-\sqrt{3}}}{2}$

b) $\cos(105°)$ $\cos(60+45)$ or $\cos\left(\frac{210}{2}\right)$

$\cos 60\cos 45 - \sin 60\sin 45$

$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

$-\sqrt{\frac{1+\cos 210}{2}}$

$-\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}$

$-\frac{\sqrt{2-\sqrt{3}}}{2}$

c) $\sin 60° + \sin 30°$

$2\sin 45\cos 15$

$2\left(\frac{\sqrt{2}}{2}\right)\left(\cos\frac{30}{2}\right)$

$\sqrt{2}\left(\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}\right)$

$\sqrt{2}\left(\frac{\sqrt{2+\sqrt{3}}}{2}\right)$

$\frac{\sqrt{2}\cdot\sqrt{2+\sqrt{3}}}{2}$

4. Write the expression as the sine, cosine, or tangent of an angle.

a) $\frac{\tan 20° - \tan 15°}{1+\tan 20°\tan 15°}$

$\tan(20-15)$

$\tan 5°$

b) $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$

$\sin\left(\frac{5\pi}{12}\right)$

c) $\cos 30°\cos 25° - \sin 30°\sin 25°$

$\cos(30+25)$

$\cos(55°)$

5. Prove the following identities.

a) $\frac{1+\tan x}{\tan x - 1} = \frac{\cot x + 1}{1 - \cot x}$

$\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}} = \frac{\frac{\cos x + \sin x}{\cancel{\cos x}}}{\frac{\sin x - \cos x}{\cancel{\cos x}}} =$

$\frac{\cos x + \sin x}{\sin x - \cos x} = \frac{\frac{\cos x}{\sin x} + \frac{\sin x}{\sin x}}{\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}} = \frac{\cot x + 1}{1 - \cot x} = \frac{\cot x + 1}{1 - \cot x}$

b) $2\sin\theta\cos^3\theta + 2\sin^3\theta\cos\theta = \sin 2\theta$

$2\sin\theta\cos\theta(\cos^2\theta + \sin^2\theta) =$

$2\sin\theta\cos\theta =$

$\sin 2\theta = \sin 2\theta$

c) $(\cos x-\sin x)^2+(\cos x+\sin x)^2=2$

$\underbrace{\cos^2x-\cancel{2\cos x\sin x}+\sin^2x}_{1}+\underbrace{\cos^2x+\cancel{2\cos x\sin x}+\sin^2x}_{1}=$

$1+1$

$2=2$

d) $\tan(x+\frac{3\pi}{4})=\frac{\tan x-1}{1+\tan x}$

$\frac{\tan x+\tan\frac{3\pi}{4}}{1-\tan x\tan\frac{3\pi}{4}}=$

$\frac{\tan x-1}{1+\tan x}=\frac{\tan x-1}{1+\tan x}$

e) $\frac{\sec^2\theta-1}{\sin\theta}=\frac{\sin\theta}{1-\sin^2\theta}$

$\frac{\tan^2\theta}{\sin\theta}=$

$\frac{\sin^2\theta}{\cos^2\theta\sin\theta}=$

$\frac{\sin\theta}{\cos^2\theta}=$

$\frac{\sin\theta}{1-\sin^2\theta}=\frac{\sin\theta}{1-\sin^2\theta}$

f) $\tan^2x-\sin^2x=\sin^2x\tan^2x$

$\frac{\sin^2x}{\cos^2x}-\frac{\sin^2x\cos^2x}{\cos^2x}=$

$\frac{\sin^2x-\sin^2x\cos^2x}{\cos^2x}=$

$\frac{\sin^2x(1-\cos^2x)}{\cos^2x}$

$\frac{\sin^2x\sin^2x}{\cos^2x}=$

$\sin^2x\tan^2x=$

$\sin^2x\tan^2x$

g) $\frac{1}{\sec x-1}+\frac{1}{\sec x+1}=2\cot x\csc x$

$\frac{\sec x+1+\sec x-1}{\sec^2x-1}=$

$\frac{2\sec x}{\tan^2x}=$

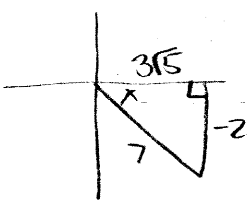
$\frac{2\cdot\frac{1}{\cos x}}{\frac{\sin^2x}{\cos^2x}}=$

$\frac{2}{\cos x}\cdot\frac{\cos^2x}{\sin^2x}=$

$\frac{2\cos x}{\sin x\sin x}=$

$2\cot x\csc x=2\cot x\csc x$

6. Find sin2x and cos2x given that sinx= $\frac{-2}{7}$ and $\frac{3\pi}{2}<x<2\pi.$



$\sin 2x=2\sin x\cos x$

$=2\left(-\frac{2}{7}\right)\left(\frac{3\sqrt5}{7}\right)$

$=-\frac{12\sqrt5}{49}$

$\cos 2x=\cos^2x-\sin^2x$

$=\left(\frac{3\sqrt5}{7}\right)^2-\left(-\frac{2}{7}\right)^2$

$=\frac{45}{49}-\frac{4}{49}$

$=\frac{41}{49}$

Ch 5 Review WS #2 Name ______________________________ Per____

Do all of your work on a separate piece of paper.

Verify each identity.

1. tanαsinα+cosα=secα
2. $(\sin^2 x)(1+\cot^2 x)=1$
3. $\sec(-\theta)-\tan(-\theta)=\frac{1+\sin\theta}{\cos\theta}$
4. $\frac{\cot^2\beta-1}{1+\cot^2\beta}=1-2\sin^2\beta$
5. $\frac{\sin\alpha}{\cos\alpha\tan\alpha}=1$
6. $\frac{\tan\theta-\cot\theta}{\tan\theta+\cot\theta}+2\cos^2\theta=1$

7-9, Find all solutions of the equation.

7. cosx = 1-cosx
8. 4sin2x=1+2sin2x
9. $\cos x=\sqrt{3}\sin x$

10-12, Find all solutions of the equation in the interval [0,2π).

10. $3-2\cos^2 x = 3\sin x$
11. $4\cot^2 x-4 = 0$
12. secxsinx = 2sinx

13-20, Find the exact value.

13. $\sin(-165^\circ)$
14. $\cos(\frac{-5\pi}{12})$
15. $\tan\frac{7\pi}{8}$
16. $\cos 105^\circ$
17. $\cos 80^\circ\cos 20^\circ + \sin 80^\circ\sin 20^\circ$
18. $\tan\theta = \frac{3}{4}, \pi < \theta < \frac{3\pi}{2}$ find $\sin 2\theta$
19. $\sec\theta = 3, \frac{3\pi}{2} < \theta < 2\pi$, find $\sin\frac{\theta}{2}$
20. $\csc\alpha = -2, \pi<\alpha<\frac{3\pi}{2}$, find $\cos\frac{\alpha}{2}$.

Answers: 7. $\frac{\pi}{3}+2n\pi, \frac{5\pi}{3}+2\pi n$ 8. $\frac{\pi}{12}+n\pi, \frac{5\pi}{12}+n\pi$ 9. $\frac{\pi}{6}+n\pi$ 10. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$
11. $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 12. $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ 13. $\frac{\sqrt{2}-\sqrt{6}}{4}$ 14. $\frac{\sqrt{6}-\sqrt{2}}{4}$ 15. $\frac{-2+\sqrt{2}}{\sqrt{2}}$ 16. $\frac{\sqrt{2}-\sqrt{6}}{4}$ 17. ½ 18. $\frac{24}{25}$
19. $\frac{\sqrt{3}}{3}$ 20. $\frac{-\sqrt{2-\sqrt{3}}}{2}$

5.3 Solving Trig Equations Practice Worksheet #1
Pre-calculus

Name: Danielle
Date: ______ Block: ____

Solve for the unknown variable on the interval $0 \le x < 2\pi$.

1. $4\cos^2 x - 3 = 0$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$\boxed{\frac{\pi}{6} \quad \frac{5\pi}{6} \quad \frac{7\pi}{6} \quad \frac{11\pi}{6}}$

2. $\sqrt{2}\sin 2x = 1$

$\sin 2x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$\boxed{\frac{\pi}{8}, \frac{3\pi}{8}}$

3. $3\cot^2 x - 1 = 0$

$\cot^2 x = \pm\frac{\sqrt{3}}{3}$

$\tan x = \pm\sqrt{3}$

$\boxed{\frac{\pi}{3} \quad \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad \frac{5\pi}{3}}$

4. $\cos^3 x = \cos x$

$\cos^3 - \cos x = 0$

$\cos x(\cos^2 - 1) = 0$

$\cos x = 0$ @ $\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$\cos x = \pm 1$ @ $\boxed{0, \pi}$

5. $\sin x - 2\sin x\cos x = 0$

$\sin x(1 - 2\cos x)$

$\sin x = 0$ @ $\boxed{0, \pi}$

$\cos x = \frac{1}{2}$ @ $\boxed{\frac{\pi}{3}, \frac{5\pi}{3}}$

6. $2\sin^2 x - \sin x - 3 = 0$

$2u^2 - u - 3$

$2u^2 - 3u + 2u - 3$

$u \qquad 1$

$(u+1)(2u-3)$

$\sin = -1$ @ $\boxed{\frac{3\pi}{2}}$ $\qquad \sin x = \frac{3}{2}$

7. $\csc^2 x - \csc x - 2 = 0$

$x^2 - x - 2$

$(x-2)(x+1)$

$\csc x = 2 \qquad \csc x = -1$

$\sin x = \frac{1}{2} \qquad \sin x = -1$

@ $\boxed{\frac{\pi}{6}, \frac{5\pi}{6}}$ @ $\boxed{\frac{3\pi}{2}}$

8. $\cos^2 x = 1 - \sin x$

$1 - \sin^2 = 1 - \sin$

$-1 + \sin \quad -1 + \sin$

$+\sin^2 \cancel{+} \sin = 0$

$\sin(\sin - 1) = 0$

$\sin = 0$ @ $\boxed{0, \pi}$ $\qquad \sin = 1$ @ $\boxed{\frac{\pi}{2}}$

Solve for the unknown variable on the given interval.

9. $\sqrt{3} + \tan(2x) = 0$ on $[0, 2\pi)$.

$\tan(2x) = -\sqrt{3}$

$\tan x = -\sqrt{3}$ @ $\frac{2\pi}{3}$

$\frac{5\pi}{3}$

$x = \boxed{\frac{2\pi}{6}, \frac{5\pi}{6}}$

10. $\cos(\pi x) = 0.5$ on $[0, 2)$.

$\cos x = \frac{1}{2}$ @

$\frac{\pi}{6}, \frac{11\pi}{6}$

$\div \pi \quad \div \pi$

$\boxed{\frac{1}{6}, \frac{11}{6}}$

11. $\sin\left(\frac{x}{2}\right) - 1 = 0$ on $[0, 8\pi)$.

$\sin x = 1$ @ $\frac{\pi}{2}$

$\cdot 2 \qquad \frac{5\pi}{2}$

$\boxed{\pi, 5\pi}$

19. $\cos 2x = 0$

$0, \pi$ so

$\boxed{0 \pm \frac{\pi}{2} n}$

$\cos x = -\frac{1}{2}$

$\boxed{x = \frac{2\pi}{3}, \frac{5\pi}{3} \pm 2\pi n}$

20. $\tan \frac{x}{3} = 1$

$\tan x = 1$ @ $\frac{\pi}{4} \cdot 3$ $\frac{5\pi}{4} \cdot 3$

so $\boxed{x = \frac{3\pi}{4} \pm 2\pi n}$

21. $\csc^2 2x - 3\csc 2x - 4 = 0$

$u^2 - 3u - 4$

$(u-4)(u+1) = 0$

$\csc 2x = 4$ $\quad\quad$ $\csc 2x = -1$

5.3 Solving Trig Equations – Worksheet #2
Pre-calculus

Name: Danielle
Date: ______________ Block: ___

Part 1: Solve for the unknown variable. Give all of the exact general solutions.

1. $\sin\theta = \frac{\sqrt{2}}{2}$

$\boxed{\frac{\pi}{4} \pm 2\pi K \quad \frac{3\pi}{4} \pm 2\pi K}$

2. $\cos\theta = \sin\theta$

$\frac{\cos\theta}{\cos\theta} = \frac{\sin\theta}{\cos\theta}$

$1 = \tan\theta$

$\boxed{\frac{\pi}{4} \pm \pi K}$

3. $\tan\theta = 1$

4. $1 + \sin\theta = 2\cos^2\theta$

$1 + \sin\theta = 2(1 - \sin^2)$

$-1 \quad -\sin$

$2 - 2\sin^2 - \sin - 1$

$+2\sin^2 + \sin + 1$

$(2\sin - 1)(\sin + 1)$

$\sin = \frac{1}{2} \qquad \sin = -1$

$@ \boxed{\frac{\pi}{6}, \frac{5\pi}{6} \pm 2\pi K} \quad \frac{3\pi}{2} \pm 2\pi K$

5. $2\cos^2\theta + \cos\theta = 0$

$\cos(2\cos + 1) = 0$

$\cos = 0 \; @ \; \frac{\pi}{2}, \frac{3\pi}{2}$

$\cos = \frac{-1}{2} \; @ \; \frac{5\pi}{6}, \frac{7\pi}{6}$

$\boxed{\frac{\pi}{2} \pm \pi K \quad \frac{5\pi}{6} \pm 2\pi K \quad \frac{7\pi}{6} \pm 2\pi K}$

6. $\sin 3\theta = -1$

$\sin = -1 \; @ \; \frac{3\pi}{2} \div 3$

$\boxed{\frac{\pi}{2} \pm 2\pi K}$

7. $\sin^2\theta - 1 = 0$

$\sin = \pm 1$

$\frac{\pi}{2}, \frac{3\pi}{2}$

$\boxed{\frac{\pi}{2} \pm \pi K}$

8. $\cos 2\theta = ½$

$\cos = \frac{1}{2}$

$@ \; \frac{\pi}{3}, \frac{5\pi}{3} \div 2$

$\boxed{\frac{\pi}{6}, \frac{5\pi}{6} \pm 2\pi K}$

9. $2\sin^2\theta - \sin\theta - 1 = 0$

$(2\sin + 1)(\sin - 1)$

$\sin = \frac{-1}{2} \; @ \; \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sin = 1 \; @ \; \frac{\pi}{2}$

10. $\tan 4\theta = -1$

$\tan = -1 \; @ \; \frac{3\pi}{4}, \frac{7\pi}{4}$

$\boxed{\frac{3\pi}{16}, \frac{7\pi}{16}} \pm 2\pi K$

11. $\tan^2 3x = 3$

$\tan(3x) = \pm\sqrt{3}$

$\frac{\pi}{3} \quad \frac{2\pi}{3} \quad \frac{4\pi}{3} \quad \frac{5\pi}{3}$

$\div 3 \qquad \pm 2\pi K \qquad \pm 2\pi K$

$\boxed{\frac{\pi}{9} \quad \frac{2\pi}{9} \quad \frac{4\pi}{9} \quad \frac{5\pi}{9}}$

12. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{2}}{2}$

$\cos = \frac{\sqrt{2}}{2}$

$@ \; \frac{\pi}{4}, \frac{7\pi}{4} \cdot 2$

$\hookrightarrow \boxed{\frac{\pi}{2} \quad \frac{7\pi}{2}}$

calc.

Part 2: Solve by approximating the solutions on the interval [0,2π).

13. $2\sin^2 x + 3\sin x + 1 = 0$

14. $4\sin^2 x = 2\cos x + 1$

$4(1-\cos^2) = 2\cos + 1$

$4 - 4\cos^2 = 2\cos + 1$

$4\cos^2 + 2\cos - 3$

15. $\csc x + \cot x = 1$

16. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

17. $\sec^2 x + 0.5\tan x = 1$

Part 3: Use the calculator's inverse trig functions to approximate the solutions. Remember that you must also find the other solution by either adding π, subtracting the value from π, or subtracting the value from 2π.

18. $\tan\theta = 4$

$\tan^{-1}(4) = \theta$

$\boxed{\begin{matrix} 1.35 \\ 4.46 \end{matrix}}$

19. $\cos\theta = 0.84$

$\cos^{-1}(.84)$

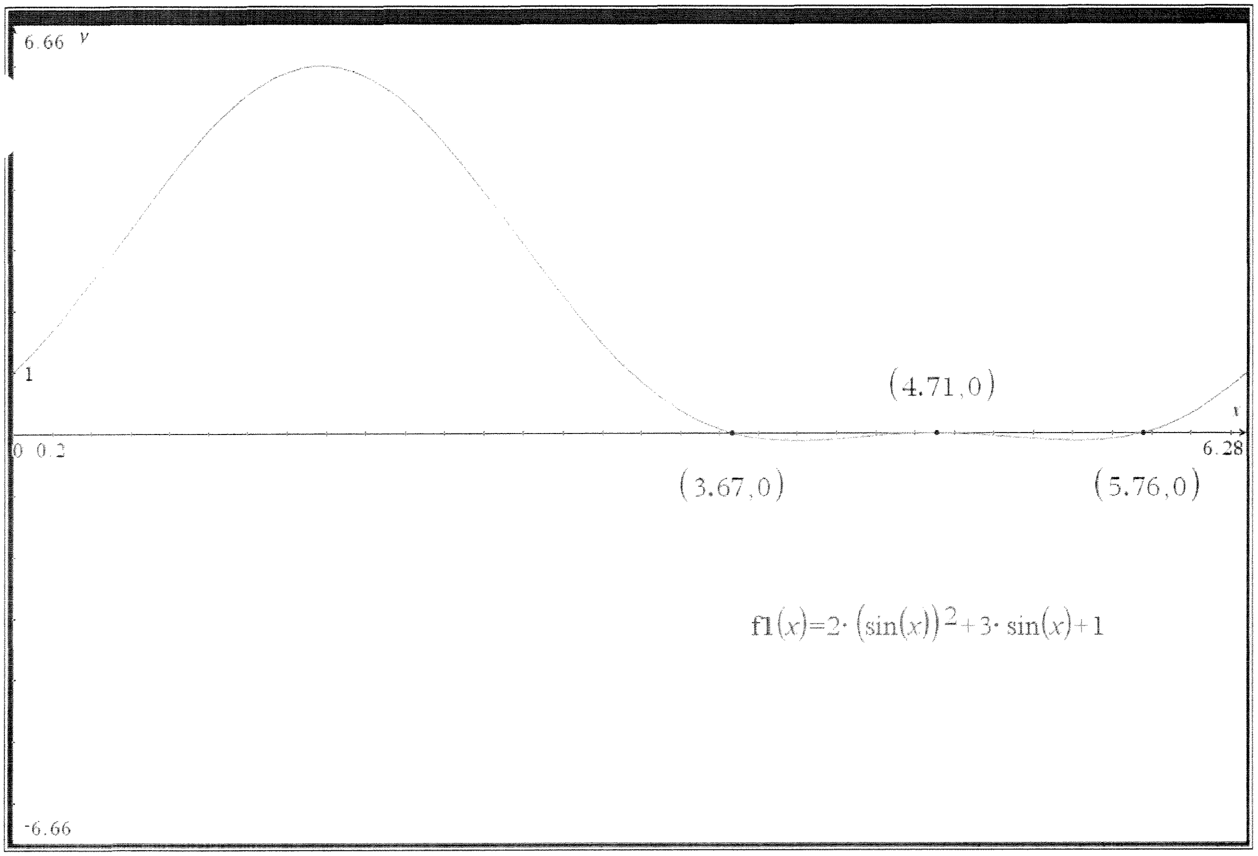
$\boxed{\begin{matrix} .57 \\ 3.715 \end{matrix}}$

20. $\sin\theta = 0.63$

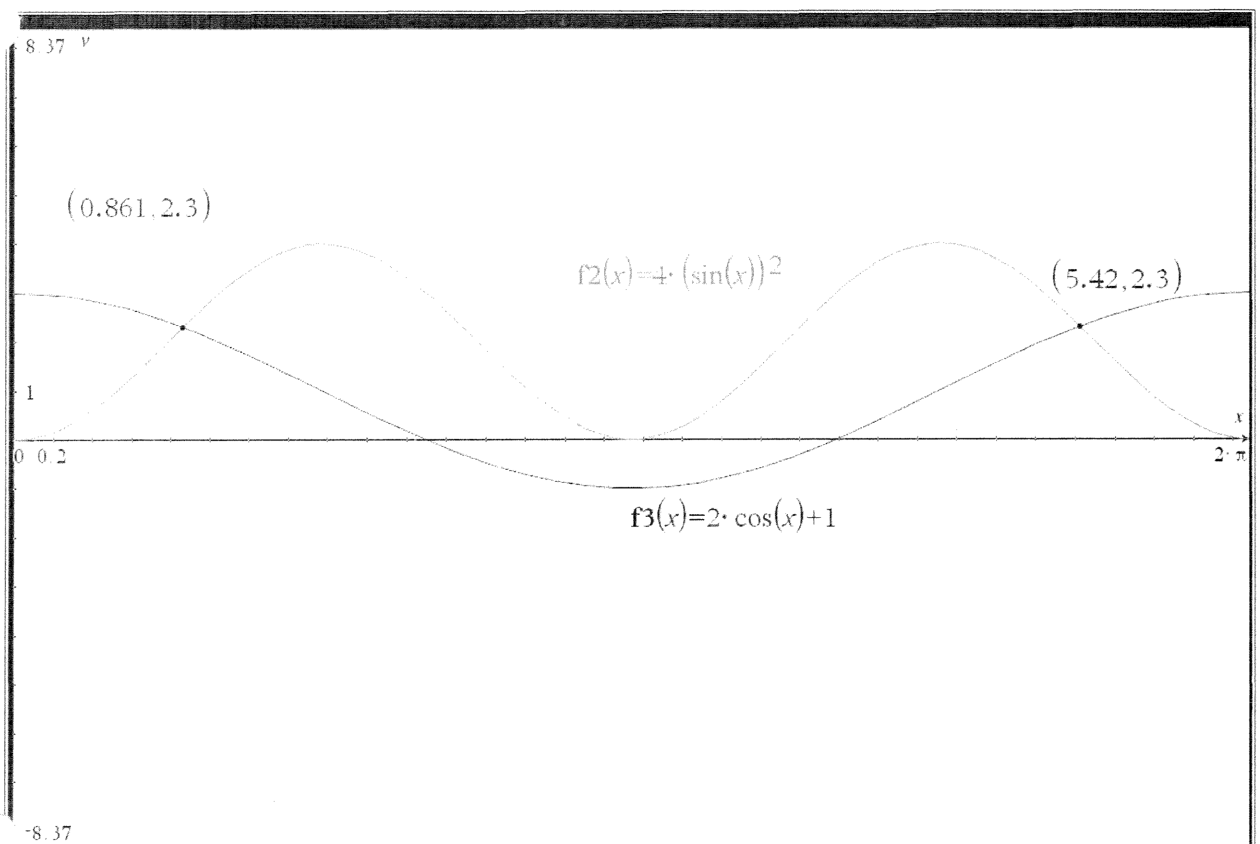
$\sin^{-1}(.63)$

$\boxed{\begin{matrix} .68 \\ 3.82 \end{matrix}}$

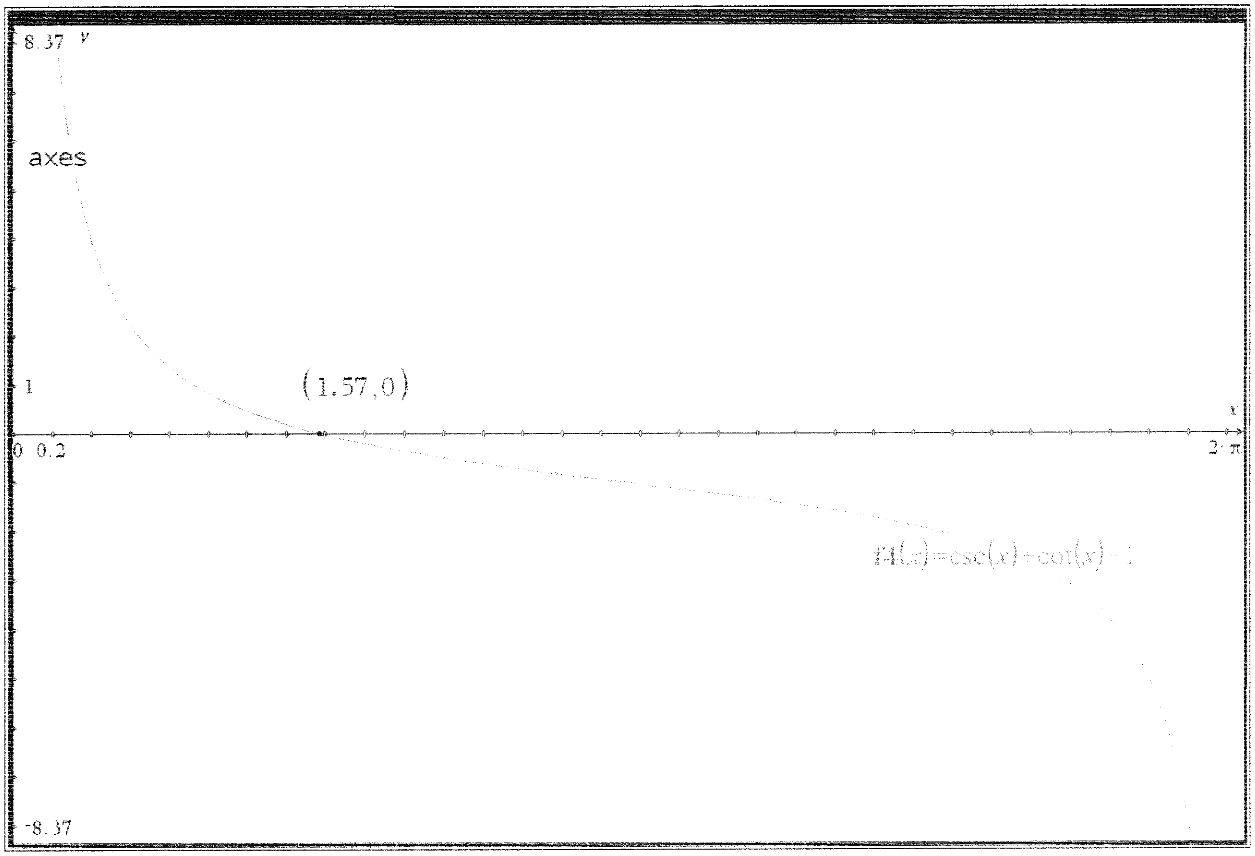
13.


6.66
y
1
(4.71,0)
x
0 0.2
6.28
(3.67,0)
(5.76,0)
f1(x)=2· (sin(x))2+3· sin(x)+1
-6.66

14.

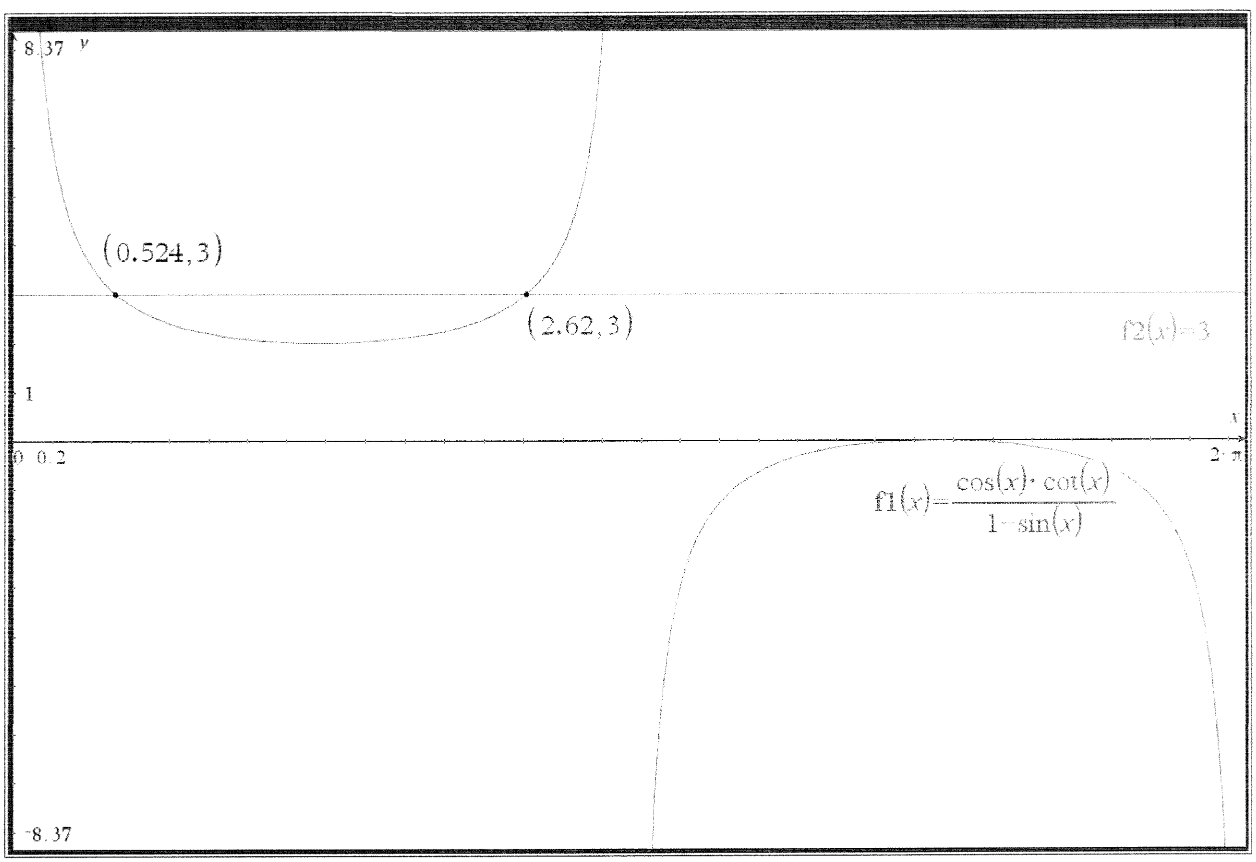

8.37
y
(0.861,2.3)
f2(x)=4· (sin(x))2
(5.42,2.3)
1
x
0 0.2
2·π
f3(x)=2· cos(x)+1
-8.37

15.



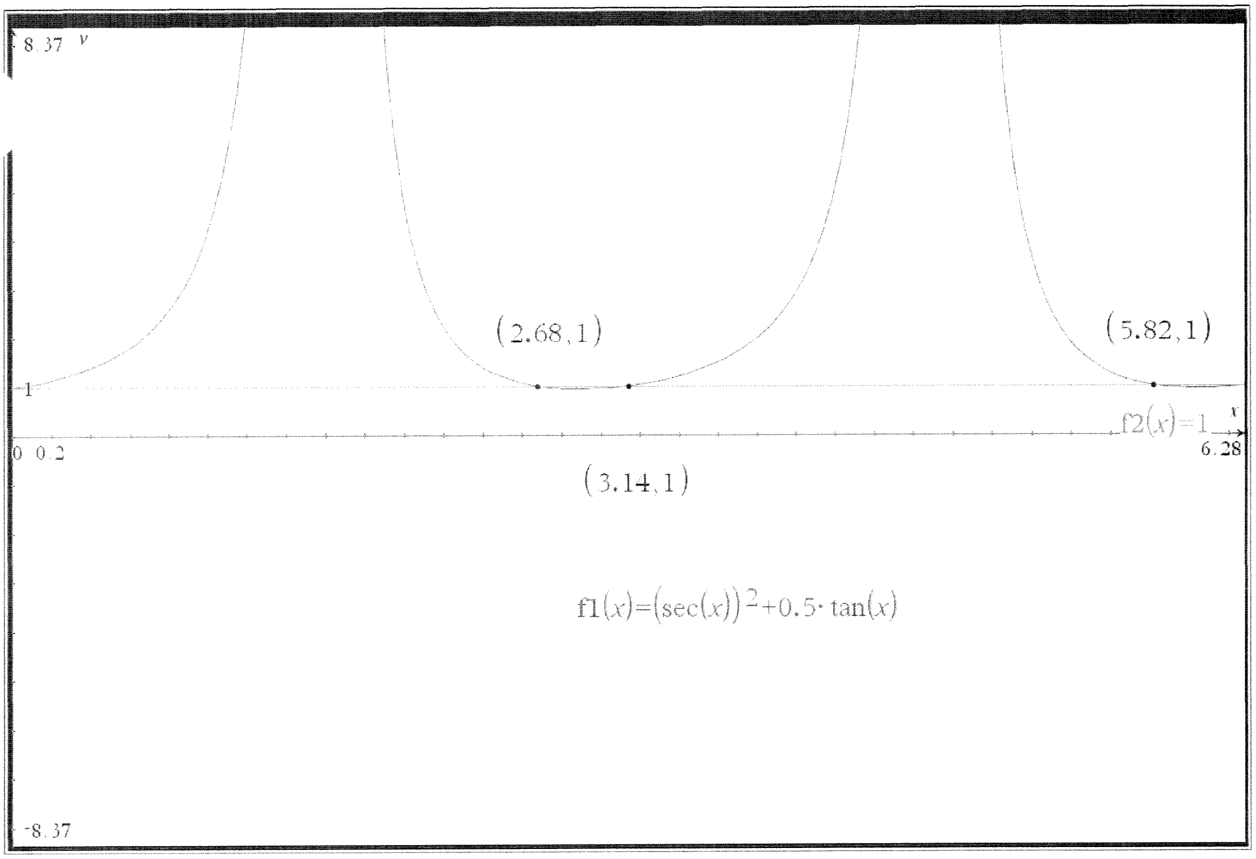
8.37
y
axes
1
(1.57,0)
x
0 0.2
2·π
f4(x)=csc(x)+cot(x)−1
-8.37

16.



8.37
y
(0.524,3)
(2.62,3)
f2(x)=3
1
x
0 0.2
2·π
f1(x)=cos(x)·cot(x)/(1−sin(x))
-8.37

17.



PreCalculus Name________________________________ ID: 1

Review A

Date________________ Period____

Find the exact value of each.

1) $\cos -\frac{\pi}{12}$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

2) $\sin \frac{5\pi}{12}$ $\frac{\sqrt{6}+\sqrt{2}}{4}$

3) $\tan 75°$

$2+\sqrt{3}$

4) $\cos \frac{11\pi}{12}$ $\frac{-\sqrt{6}-\sqrt{2}}{4}$

Find the exact value of each expression.

5) $\sin \theta = \frac{2\sqrt{105}}{21}$ and $\frac{\pi}{2} < \theta < \pi$

Find $\cos 2\theta$

$-\frac{19}{21}$

6) $\cos \theta = -\frac{12}{13}$ and $90° < \theta < 180°$ $\frac{5\sqrt{26}}{26}$

Find $\sin \frac{\theta}{2}$

7) $\cos \theta = \frac{24}{25}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\cos \frac{\theta}{2}$

$-\frac{7\sqrt{2}}{10}$

8) $\tan \theta = \frac{3}{4}$ and $0° < \theta < 90°$ $\frac{\sqrt{10}}{10}$

Find $\sin \frac{\theta}{2}$

Solve each equation for $0 \le \theta < 2\pi$.

9) $-3 + 3\cot \frac{\theta}{3} = -4 + 2\cot \frac{\theta}{3}$

No solution.

10) $-3 - 9\sin\left(\theta + \frac{\pi}{4}\right) = -6\sqrt{3} - 3$

No solution.

11) $4\sqrt{3} + 3 = 3 - 8\cos\left(\theta + \frac{3\pi}{2}\right)$

$\left\{\frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

12) $-4 - 5\csc 4\theta = -3\csc 4\theta$

$\left\{\frac{7\pi}{24}, \frac{11\pi}{24}, \frac{19\pi}{24}, \frac{23\pi}{24}, \frac{31\pi}{24}, \frac{35\pi}{24}, \frac{43\pi}{24}, \frac{47\pi}{24}\right\}$

Solve each equation for all solutions.

13) $\sin 2x - \cos x - 2\sin x + 1 = 0$

14) $2 \cdot \cos^2 x + \cos x - 1 = 0$

15) $\sin x + 2\cos x = 1$

16) $\cos 4x - \cos 6x = 0$

Use a graphing utility to solve each equation on the interval $(0,2\pi)$. Round to two decimal places.

17) $2x = 5\sin x$

18) $2\sin x + 3\cos x = 4x$

Verify the identity.

19) $1 - 8 \cdot \sin^2 x \cdot \cos^2 x = \cos 4x$

20) $\frac{(2 \cdot \sin^2 x - 1)^2}{\sin^4 x - \cos^4 x} = 1 - 2 \cdot \cos^2 x$

Find the exact value.

21) $\sin\left(cos^{-1} \cdot \frac{5}{13} - cos^{-1} \cdot \frac{4}{5}\right)$

PreCalculus | Name________________________ ID: 1

Review B

Date________________ Period____

Find the exact value of each.

1) $\sin \frac{\pi}{12}$ $\frac{\sqrt{6}-\sqrt{2}}{4}$

2) $\sin -75°$

$\frac{-\sqrt{6}-\sqrt{2}}{4}$

3) $\tan 285°$

$-2-\sqrt{3}$

4) $\cos 285°$ $\frac{\sqrt{6}-\sqrt{2}}{4}$

Find the exact value of each expression.

5) $\cos \theta = -\frac{\sqrt{26}}{26}$ and $\frac{\pi}{2} < \theta < \pi$ $\frac{5}{12}$

Find $\tan 2\theta$

6) $\cos \theta = \frac{15}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\tan \frac{\theta}{2}$

$-\frac{1}{4}$

7) $\cos \theta = \frac{15}{17}$ and $\frac{3\pi}{2} < \theta < 2\pi$ $-\frac{4\sqrt{17}}{17}$

Find $\cos \frac{\theta}{2}$

8) $\sin \theta = -\frac{3}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$

Find $\cos 2\theta$

$\frac{7}{25}$

Solve each equation for $0 \le \theta < 2\pi$.

9) $2 - 2\cos \frac{\theta}{2} = 5 - 8\cos \frac{\theta}{2}$

$\left\{\frac{2\pi}{3}\right\}$

10) $\frac{5}{4} + \tan -3\theta = 1 + \frac{3}{4} \cdot \tan -3\theta$

$\left\{\frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}\right\}$

11) $2 + 7\cos\dfrac{\theta}{2} = 4\sqrt{2} + 2 - \cos\dfrac{\theta}{2}$

$\left(\dfrac{\pi}{2}\right)$

12) $5 - 3\tan\dfrac{\theta}{2} = 4 - 4\tan\dfrac{\theta}{2}$

$\left(\dfrac{3\pi}{2}\right)$

Solve each equation for all solutions.

13) $\sec^2 2x + \tan 2x = 0$

14) $\sin x + \cos x = -\sqrt{2}$

15) $\sin^2 x - \sin x - 1 = 0$

16) $\cos 2x + \cos 4x = 0$

Use a graphing utility to solve each equation on the interval (0,2π). Round to two decimal places.

17) $x = \sin x - \cos x$

18) $x^2 - 2\sin 2x = 3x$

Verify the identity.

19) $1 - \dfrac{\cos^2 x}{1 + \sin x} = \sin x$

20) $4 \cdot \sin^2 x + 2 \cdot \cos^2 x = 4 - 2 \cdot \cos^2 x$

Find the exact value $\sin \dfrac{x}{2}$.

21) $\sin x = -\dfrac{3}{5}, \pi < x < 3\pi/2$

Answers to Review A (ID: 1)

1) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

2) $\sqrt{3}-2$

3) $\dfrac{-\sqrt{6}-\sqrt{2}}{4}$

4) $\dfrac{\sqrt{2}-\sqrt{6}}{4}$

5) $\dfrac{5}{3}$

6) $-\dfrac{2\sqrt{13}}{13}$

7) $-\dfrac{24}{25}$

8) $\dfrac{1}{2}$

9) $\left\{\dfrac{5\pi}{6}, \dfrac{11\pi}{6}\right\}$

10) $\left\{\dfrac{3\pi}{2}\right\}$

11) $\left\{\dfrac{5\pi}{6}, \dfrac{11\pi}{6}\right\}$

12) $\left\{\dfrac{\pi}{12}, \dfrac{19\pi}{12}\right\}$

13) $\left\{\dfrac{7\pi}{12}, \dfrac{13\pi}{12}\right\}$

14) No solution.

15) $\{\pi n\}$

0 ± πn

16) $\left\{\dfrac{\pi}{3}+\pi n\right\}$

17) $\left\{\dfrac{3\pi}{4}+2\pi n, \dfrac{\pi}{4}+2\pi n\right\}$

18) $\left\{\dfrac{\pi}{6}+\pi n\right\}$

19) $\left\{\dfrac{3\pi}{4}+2\pi n, \dfrac{5\pi}{4}+2\pi n\right\}$

20) $\left\{\dfrac{5\pi}{3}+\pi n\right\}$

21) $\dfrac{5\pi}{6}$

22) $\dfrac{\pi}{4}$

Answers to Review B (ID: 1)

1) $\dfrac{2\sqrt{14}}{5}$ and $-\dfrac{2\sqrt{14}}{9}$ 2) $-\sqrt{3}$ and $\dfrac{\sqrt{3}}{2}$ 3) $\dfrac{2\sqrt{5}}{5}$ and $\sqrt{5}$ 4) $-\dfrac{\sqrt{3}}{2}$ and $-\dfrac{2\sqrt{3}}{3}$

5) $\dfrac{\sec x}{\cos x + \sec x}$ Decompose into sine and cosine

$\dfrac{\dfrac{1}{\cos x}}{\cos x + \dfrac{1}{\cos x}}$ Simplify

$\dfrac{1}{1 + \cos^2 x}$ ■

6) $\tan^2 x - \sec^2 x \sin x$ Decompose into sine and cosine

$\left(\dfrac{\sin x}{\cos x}\right)^2 - \left(\dfrac{1}{\cos x}\right)^2 \sin x$ Simplify

$\dfrac{\sin x \cdot (\sin x - 1)}{\cos^2 x}$ ■

7) $\sec^2 x(\csc^2 x - 1)$ Use $\cot^2 x + 1 = \csc^2 x$

$\sec^2 x \cot^2 x$ Decompose into sine and cosine

$\left(\dfrac{1}{\cos x}\right)^2 \cdot \left(\dfrac{\cos x}{\sin x}\right)^2$ Simplify

$\dfrac{1}{\sin^2 x}$ Use $\csc x = \dfrac{1}{\sin x}$

$\csc^2 x$ ■

8) $-\sin^2 x \sec^2 x$ Use $\sec x = \dfrac{1}{\cos x}$

$-\dfrac{\sin^2 x}{\cos^2 x}$ Use $\tan x = \dfrac{\sin x}{\cos x}$

$-\tan^2 x$ Use $\tan^2 x + 1 = \sec^2 x$

$1 - \sec^2 x$ ■

9) $\left\{\dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$ 10) $\left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}$

11) $\left\{0, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}\right\}$ 12) $\{0\}$ 13) $\left\{\dfrac{\pi}{3}, \dfrac{5\pi}{3}\right\}$ 14) $\left\{\dfrac{\pi}{4}, \dfrac{7\pi}{4}\right\}$

15) $\left\{\dfrac{\pi}{4}, \dfrac{5\pi}{4}\right\}$ 16) $\left\{\dfrac{\pi}{6}, \dfrac{\pi}{2}, \dfrac{5\pi}{6}\right\}$ 17) $\left\{\dfrac{\pi}{3}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{5\pi}{3}\right\}$ 18) $\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}$

19) $\tan(45 - \theta)$

$$= \frac{\tan 45 - \tan \theta}{1 + \tan 45 \tan \theta}$$

$$= \frac{1 - \tan \theta}{1 + \tan \theta}$$

$$= \frac{1 - \tan \theta}{1 + \tan \theta}$$

20) $\cos\left(\theta + \frac{3\pi}{2}\right)$

$$= \cos \theta \cos \frac{3\pi}{2} - \sin \theta \sin \frac{3\pi}{2}$$

$$= \cos \theta \cdot 0 - \sin \theta \cdot -1$$

$$= \sin \theta$$

PreCalc Ch 5 Review WS #1 Name________________________________ Per _______

1. Find all solutions to the equation in the interval [0,2π).

a) $4cos^2x = 3$

b)$sinxtan^2x = sinx$

c) $\sin\left(x+\frac{\pi}{6}\right) - \sin\left(x-\frac{\pi}{6}\right) = \frac{1}{2}$

2. Find all solutions to the equation, using your calculator in the interval [0,2π). Round to two decimal places.

a) cos2x+5cosx=2

b) sin3x=sinx

c) 5sin2x-12cos2x=0

3. Find the exact value.

a) $\sin\frac{\pi}{12}$

b) cos(105°)

c) sin60°+sin30°

4. Write the expression as the sine, cosine, or tangent of an angle.

a) $\frac{tan20^o - tan15^o}{1+tan20^o tan15^o}$

b) $\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$

c) cos30°cos25°-sin30°sin25°

5. Prove the following identities.

a) $\frac{1+tanx}{tanx-1} = \frac{cotx+1}{1-cotx}$

b) $2\sin\theta\cos^3\theta + 2\sin^3\theta\cos\theta = \sin 2\theta$

c) $(\cos x-\sin x)^2 + (\cos x + \sin x)^2 = 2$

d) $\tan(x+\frac{3\pi}{4}) = \frac{\tan x-1}{1+\tan x}$

e) $\frac{\sec^2\theta-1}{\sin\theta} = \frac{\sin\theta}{1-\sin^2\theta}$

f) $\tan^2 x - \sin^2 x = \sin^2 x\tan^2 x$

g) $\frac{1}{\sec x-1} + \frac{1}{\sec x+1} = 2\cot x\csc x$

6. Find sin2x and cos2x given that sinx= $\frac{-2}{7}$ and $\frac{3\pi}{2} < x < 2\pi.$